

1. Opera y simplifica:

$$a) \frac{(-27)^3 \cdot 32^{-5} \cdot (-8)^5 \cdot (25^2)^{-6}}{(-72)^4 \cdot (-50^3)^4} = \frac{(3^3)^3 \cdot (2^5)^{-5} \cdot (2^3)^5 \cdot (5^4)^{-6}}{(3^2 \cdot 2^3)^4 \cdot [(5^2 \cdot 2)^3]^4} = \frac{3^9 \cdot 2^{-25} \cdot 2^{15} \cdot 5^{-24}}{3^8 \cdot 2^{12} \cdot 5^{24} \cdot 2^{12}} = \frac{3}{2^{34} \cdot 5^{48}}$$

$$b) \frac{5}{1 - \frac{1}{1 + \frac{1}{1 - \frac{1}{5}}}} = \frac{5}{1 - \frac{1}{1 + \frac{5}{4}}} = \frac{5}{1 - \frac{4}{9}} = \frac{5}{\frac{5}{9}} = 9$$

2. Sean los intervalos  $A = \{x/x \in \mathbb{R}, 3 < x < 5\}$ ;  $B = \{x/x \in \mathbb{R} - 2 \leq x \leq 5\}$

a) Expresa en forma de intervalo los conjuntos A y B

b) Halla  $A \cup B$  y  $A \cap B$

Solución:

$$a) A = (3, 5); A = [-2, 5]$$

$$b) A \cup B = [-2, 5]; A \cap B = (3, 5)$$

3. Opera y simplifica:

$$a) \sqrt{256x^2y} + \frac{1}{3}\sqrt[4]{\frac{81y^2}{x^{-4}}} - x\sqrt{225y} = x \cdot \sqrt{2^8y} + \frac{3}{3}x \cdot \sqrt[4]{y^2} - x \cdot \sqrt{3^2 \cdot 5^2y} = 16x \cdot \sqrt{y} + x \cdot \sqrt{y} - 15x \cdot \sqrt{y} = 2x \cdot \sqrt{y}$$

$$b) \frac{1-2\sqrt{5}}{1+2\sqrt{5}} = \frac{1-2\sqrt{5}}{1+2\sqrt{5}} \cdot \frac{1-2\sqrt{5}}{1-2\sqrt{5}} = \frac{(1-2\sqrt{5})^2}{1^2 - (2\sqrt{5})^2} = \frac{1-2 \cdot 2\sqrt{5} + (2\sqrt{5})^2}{1-4 \cdot 5} = \frac{1-4\sqrt{5}+4 \cdot 5}{-19} = \frac{21-4\sqrt{5}}{-19}$$

$$c) \frac{\sqrt[3]{2a^2\sqrt{a^{-2}}}}{2\sqrt{a}} = \frac{\sqrt[3]{\sqrt{(2a^2)^2}a^{-2}}}{\sqrt{2^2a}} = \frac{\sqrt[3]{\sqrt{4a^4}a^{-2}}}{\sqrt{4a}} = \frac{\sqrt[6]{4a^2}}{\sqrt{4a}} = \sqrt[6]{\frac{4a^2}{(4a)^3}} = \sqrt[6]{\frac{4a^2}{4^3a^3}} = \sqrt[6]{\frac{1}{16a}}$$

4. Opera y simplifica:

$$\frac{\log_2 \sqrt[5]{8} + \log_2 16 + \log_2 \frac{1}{8}}{2\log_2 4 - 3\log_2 2} = \frac{\log_2 (2^3)^{\frac{1}{5}} + \log_2 2^4 + \log_2 2^{-3}}{\log_2 4^2 - \log_2 2^3} = \frac{\log_2 2^{\frac{3}{5}} + 4\log_2 2 - 3\log_2 2}{4\log_2 2 - 3\log_2 2} = \frac{\frac{3}{5} + 4 - 3}{4 - 3} = \frac{8}{5}$$

5. Aplica las propiedades de los logaritmos para escribir la siguiente expresión como un solo logaritmo:

$$\log_2(xy) - \log_2\left(\frac{x}{y^2}\right) + \log_2\left(\frac{x^2y}{2}\right) = \log_2\left(\frac{xy}{\frac{x}{y^2}}\right) + \log_2\left(\frac{x^2y}{2}\right) = \log_2\left(y^3\left(\frac{x^2y}{2}\right)\right) = \log_2\left(\frac{x^2y^4}{2}\right)$$

6. Halla el valor de  $x$  en cada caso:

a)  $\log_{32} 2 = x \Rightarrow 32^x = 2 \Rightarrow 2^{5x} = 2 \Rightarrow 5x = 1 \Rightarrow x = \frac{1}{5}$

b)  $\log_9 \frac{1}{27} = x \Rightarrow 9^x = \frac{1}{27} \Rightarrow 3^{2x} = 3^{-3} \Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$

c)  $\log_x 2 = \frac{1}{5} \Rightarrow x^{\frac{1}{5}} = 2 \Rightarrow \left(x^{\frac{1}{5}}\right)^5 = 2^5 \Rightarrow x^{\frac{5}{5}} = 2^5 \Rightarrow x = 32$

d)  $\log_{\frac{1}{2}} x = 3 \Rightarrow \left(\frac{1}{2}\right)^3 = x \Rightarrow x = \frac{1}{8}$

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